Closing Tue: 13.3(part 2), 13.4
Closing Thu: 14.1, 14.3(part 1)
Midterm 1 will be returned Tuesday.

13.4 Position, Velocity, Acceleration

IF *t = time*, then
$\vec{\boldsymbol{r}}(t) = < x(t), y(t), z(t) >=$ position,

And since

$$\vec{\boldsymbol{r}}'(t) = \lim_{h\to 0} \frac{\vec{\boldsymbol{r}}(t+h) - \vec{\boldsymbol{r}}(t)}{h} = \frac{\text{change in position}}{\text{change in time}}$$

we have
$\vec{\boldsymbol{r}}'(t) = \vec{\boldsymbol{v}}(t) = < x'(t), y'(t), z'(t) >=$ velocity
$|\vec{\boldsymbol{r}}'(\boldsymbol{t})| = |\vec{\boldsymbol{v}}(t)| = \frac{\text{change in dist}}{\text{change in time}} =$ speed.

And since

$$\vec{\boldsymbol{r}}''(t) = \lim_{h\to 0} \frac{\vec{\boldsymbol{r}}'(t+h) - \vec{\boldsymbol{r}}'(t)}{h} = \frac{\text{change in velocity}}{\text{change in time}}$$

so we have
$\vec{\boldsymbol{r}}''(t) = \vec{\boldsymbol{a}}(t) =$ acceleration

Entry Task:
Let *t* be time in seconds and assume the position of an object (in feet) is given by

$$\vec{\boldsymbol{r}}(t) = < t, 2e^{-t} >$$

Find the following:

1. General formulas for velocity, speed and acceleration at time *t* seconds.
2. Find and illustrate the velocity, speed and acceleration at time *t* = 0 seconds.
3. What happens to velocity, speed and acceleration as *t* gets larger?

HUGE application: How to study motion.

Newton's 2nd Law of Motion states

Force = mass · acceleration

$$\vec{\boldsymbol{F}} = m \cdot \vec{\boldsymbol{a}}$$

If $\vec{\boldsymbol{F}} = \langle 0,0,0 \rangle$, then all the forces on the object 'balance out' and the object has no acceleration.

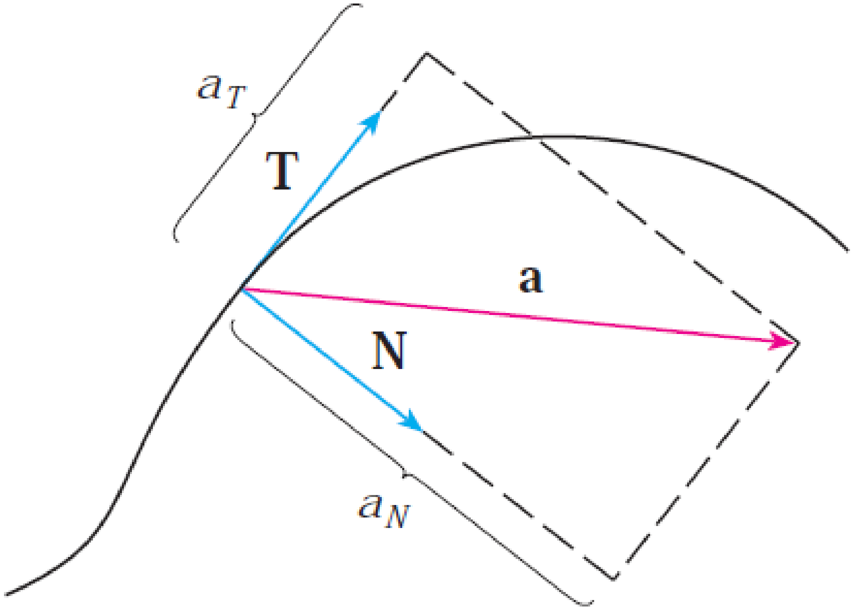
(The velocity of the object will be constant)

If $\vec{\boldsymbol{F}} \neq \langle 0,0,0 \rangle$, then acceleration will occur, and we can integrate (or solve differential equations) to find the velocity and position.

Example:
A ball with mass $m = 0.8$ kg is thrown northward into the air with initial speed of 30 m/sec at an angle of 30 degrees with the ground. A west wind applies a steady force of 4 N on the ball (west to east).
If you are standing on level ground, where does the ball land?

1. Forces?
2. Get acceleration.
3. Integrate to get $\vec{\boldsymbol{v}}(t)$
   (initial conditions?)
4. Integrate again to get $\vec{\boldsymbol{r}}(t)$
   (initial conditions?)

Measuring and describing acceleration



Recall: $comp_{\vec{b}}(\vec{\boldsymbol{a}}) = \frac{\vec{\boldsymbol{a}}\cdot\vec{\boldsymbol{b}}}{\vec{\boldsymbol{b}}}$ = projection length.

We define:

$a_T = comp_{\vec{T}}(\vec{\boldsymbol{a}}) = \vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{T}}$ = tangential comp.

$a_N = comp_{\vec{N}}(\vec{\boldsymbol{a}}) = \vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{N}}$ = normal component

Note that: $\vec{\boldsymbol{a}} = a_T\vec{\boldsymbol{T}} + a_N\vec{\boldsymbol{N}}$

Some helpful interpretations: Let's rewrite all our definitions from 13.3 in terms of velocity and acceleration and see what happens.

For ease of writing, let $\nu(t) = |\vec{\boldsymbol{v}}(t)|$ = speed

1. Since $\vec{\boldsymbol{T}}(t) = \frac{\vec{\boldsymbol{r}}'(t)}{|\vec{\boldsymbol{r}}'(t)|} = \frac{\vec{\boldsymbol{v}}(t)}{\nu(t)}$, we get $\vec{\boldsymbol{v}} = \nu\vec{\boldsymbol{T}}$.
2. Since $\kappa(t) = \frac{|\vec{\boldsymbol{T}}'(t)|}{|\vec{\boldsymbol{r}}'(t)|} = \frac{|\vec{\boldsymbol{T}}'|}{\nu(t)}$,we get $|\vec{\boldsymbol{T}}'| = \kappa\nu$.
3. Since $\vec{\boldsymbol{N}}(t) = \frac{\vec{\boldsymbol{T}}'(t)}{|\vec{\boldsymbol{T}}'(t)|} = \frac{\vec{\boldsymbol{T}}'}{\kappa\nu}$,we get $\vec{\boldsymbol{T}}' = \kappa\nu\vec{\boldsymbol{N}}$.

Differentiating the first fact above gives

$$\vec{\boldsymbol{a}} = \vec{\boldsymbol{v}}' = \nu'\vec{\boldsymbol{T}} + \nu\vec{\boldsymbol{T}}', \text{ so}$$

$$\vec{\boldsymbol{a}} = \vec{\boldsymbol{v}}' = \nu'\vec{\boldsymbol{T}} + k\nu^2\vec{\boldsymbol{N}} = a_T\vec{\boldsymbol{T}} + a_N\vec{\boldsymbol{N}}.$$

Conclusion

$$a_T = \nu' = \frac{d}{dt}|r'(t)| = \text{derivative of speed}$$

$$a_N = k\nu^2 = \text{curvature} \cdot (\text{speed})^2$$

For computational purposes, we use

$$a_T = \frac{\vec{\boldsymbol{r}}' \cdot \vec{\boldsymbol{r}}''}{|\vec{\boldsymbol{r}}'|} \quad \text{and} \quad a_T = \frac{|\vec{\boldsymbol{r}}' \times \vec{\boldsymbol{r}}''|}{|\vec{\boldsymbol{r}}'|}$$

(why do these follow from everything else on this page?)

Example:

$$\vec{\boldsymbol{r}}(t) = < \cos(t)\,, \sin(t)\,, t >$$

Find the tangential and normal components of acceleration.